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An approach to solve Slavnov-Taylor identity in D4 $\mathcal{N} = 1$ supergravity

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Abstract

We consider a particular solution to Slavnov-Taylor identity in four-dimensional supergravity. The consideration is performed for pure supergravity, no matter superfields are included. The solution is obtained by inserting dressing functions into ghost part of the classical action for supergravity. As a consequence, physical part of the effective action is local invariant with respect to diffeomorphism and structure groups of transformation for dressed effective superfields of vielbein and spin connection.

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The classical action of D4 supergravity is of the form

$$S_{\rm cl} = -\frac{6}{\kappa^2}\int\ d^4y\ d^2\Theta\ \mathcal{E}R,$$

where $\mathcal{E}$ is chiral superdensity. We use the notation of Ref. [1]. Originally in the effective action there are two local symmetries. First one is related to diffeomorphism group and second symmetry is related to the structure group of rotations in the tangent space. In case of present consideration it is $SL(2,C)$ group. In general, both the chiral density and the chiral superfield R are functions of vielbein ${E_M}^A$ and spin connection ${\phi_{MA}}^B$,

$$S_{\rm cl} = -\frac{6}{\kappa^2}\int\ d^4y\ d^2\Theta\ \mathcal{E}({E_M}^A, {\phi_{MA}}^B)R({E_M}^A, {\phi_{MA}}^B). \tag{1}$$

Usually, the chiral superfield R is taken as an independent superfield at the classical level. However here we keep it as a function of the spin connection and the vielbein that will be variables of integration in the path integral. The relation between differential 2-form of the torsion and 1-form of vielbein is

$$T = \mathcal{D}E, \tag{2}$$

where $\mathcal{D}$ is a covariant derivative $\mathcal{D} = d+\phi$ with respect to structure group with spin connection inside. This relation will be kept in the path integral by using a Lagrange multiplier π^ω, that is

$$\int\ d\pi^\omega\ \exp\int\ d^8z\ i\ E\ \pi^\omega \mathcal{C}^\omega,$$

where $\mathcal{C}^\omega$ is the constraint (2), and ω are indices of representation in the constraint (2). The coordinate z is a general coordinate of the supermanifold $z^M = (x^M, \theta^\mu, \bar\theta^{\dot\mu})$. The Grassmannian coordinate of the manifold does not coincide with the coordinate in the chiral measure. The coordinate of the chiral measure Θ is a function of the manifold coordinates and of the vielbein. The torsion T satisfies constraints in the tangent space to have flat supersymmetry as a limiting case in which curvature is absent.

Total action including gauge fixing, FP ghost action, Lagrange multiplier at the classical level can be written as

$$\begin{aligned}
S = S_{\rm cl} + S_{\rm gf} + S_{\rm gh} + \int\ d^8z\ E\ \pi^\omega\mathcal{C}^\omega \\
= -\frac{6}{\kappa^2}\int\ d^4y\ d^2\Theta\ \mathcal{E}({E_M}^A, {\phi_{MA}}^B)R({E_M}^A, {\phi_{MA}}^B) - \int\ d^8z\ \frac{1}{2\alpha}\left(\partial_M {E_A}^M\right)^2 \\
-\int\ d^8z\ {\rm Tr}\left(\frac{1}{\beta}\left[\partial_M\phi^M(z)\right]^2\right) - \int\ d^8z\ i\ b^A\ \partial_M\ \mathcal{L}_c {E_A}^M \\
-\int\ d^8z\ 2\ {\rm Tr}\ \Big(\ i\ b'(z)\partial_M\ \mathcal{D}^M\ c'(z)\Big) + \int\ d^8z\ E\ \pi^\omega\mathcal{C}^\omega.
\end{aligned} \tag{3}$$

Here $E = \det {E_M}^A$ and $\mathcal{L}_c$ is Lie derivative of the vielbein. It acts on any superfield with world index as

$$\mathcal{L}_c {E_A}^M = c^L\partial_L {E_A}^M - (\partial_L\ c^M){E_A}^L,$$

c^L here is the ghost superfield, b^A is antighost superfield. Such a choice of the gauge fixing and the ghost terms means that we fix the gauge arbitrariness by imposing the condition

$$\partial_M E_A{}^M = F_A, \quad \partial_M \phi^M{}_A{}^B = f_A{}^B(z),$$

where F_A, $f_A{}^B(z)$ are some functions. The first gauge fixing condition is to fix the gauge freedom in diffeomorphism group while second one is to fix structure group freedom.

The second gauge fixing term and the second ghost term can be made invariant with respect to the diffeomorphism group by construction. Indeed, the gauge fixing term can be made invariant by amounting the gauge fixing parameter β to superfield with property of the density E under diffeomorphism transformations. The same property can be required for antighost b'. The first gauge fixing term and the first ghost term in the action (3) are invariant with respect to structure group since the covariant derivative of the vielbein is

$$\mathcal{D}_M E_A{}^M = \left(\partial_M \delta_A^B + \phi_{MA}{}^B\right)\ E_B{}^M = \partial_M\ E_A{}^M + \phi_{BA}{}^B.$$

The l.h.s. of this equation and the second term on the r.h.s. are covariant with respect to structure group, hence the first term on the r.h.s. is also covariant. The first ghost term is covariant with respect to structure group since the Lie derivative can be written in a covariant way with respect to structure group form,

$$\mathcal{L}_c E_A{}^M = \mathcal{D}^M\ c_A + L_A{}^B E_B{}^M,$$

where $L_A{}^B$ is a matrix that takes values in the algebra of Lorentz group. The reason for the covariance is the same as in the example above. To make use of the diffeomorphism symmetry, we define the path integral extended by the dependence on the following external sources

$$\begin{aligned} Z[I, J,\ \eta,\ \rho,\ K,\ K'',\ L] = \int\ d\phi_{MA}{}^B\ dE_M{}^A\ dc^L\ db^A\ d\pi^\omega\ db'\ dc'\ \exp i\,[S \\ + \int\ d^8z\ I_M{}^A\ E_A{}^M + \int\ d^8z\ J_M{}^A{}_B\ \phi^M{}_A{}^B + i\int\ d^8z\ \eta^L c_L + i\int\ d^8z\ \rho^A b_A \\ + i\int\ d^8z\ K_M{}^A \mathcal{L}_c E_A{}^M + i\int\ d^8z\ K''_M{}^A{}_B \mathcal{L}_c\ \phi^M{}_A{}^B + \int\ d^8z\ L_M \mathcal{L}_c c^M \Big], \end{aligned} \tag{4}$$

where new external sources $K_M{}^A$, $K''_M{}^A{}_B$ and L_M coupled to the BRST variations of the vielbein, the spin connection and the ghost under group of diffeomorphisms are introduced. The action (3) is invariant with respect to BRST transformations [3],

$$\begin{aligned} E_A{}^M &\to E_A{}^M + i\mathcal{L}_c E_A{}^M \varepsilon, \\ \phi^M{}_A{}^B &\to \phi^M{}_A{}^B + i\mathcal{L}_c \phi^M{}_A{}^B \varepsilon, \\ c &\to c - \frac{1}{2}\mathcal{L}_c c\varepsilon, \\ b_A &\to b_A + \frac{1}{\alpha}\left(\partial_M E_A{}^M\right)\varepsilon. \end{aligned} \tag{5}$$

This symmetry exists due to the property of Lie derivative for any three world vectors

$$\mathcal{L}_\xi \mathcal{L}_\psi \chi^M + \mathcal{L}_\chi \mathcal{L}_\xi \psi^M + \mathcal{L}_\psi \mathcal{L}_\chi \xi^M = 0,$$

where

$$\mathcal{L}_\xi \eta^M = \xi^N(\partial_N \eta^M) - (\partial_N \xi^M)\eta^N.$$

The effective action Γ is related to $W = i\ ln\ Z$ by the Legendre transformation

$$E_A{}^M \equiv -\frac{\delta W}{\delta I_M{}^A}, \quad \phi^M{}_A{}^B \equiv -\frac{\delta W}{\delta J_M{}^A{}_B}, \quad ic^L \equiv -\frac{\delta W}{\delta \eta_L}, \quad ib^A \equiv -\frac{\delta W}{\delta \rho_A}, \tag{6}$$
$$\Gamma = -W - \int d^8z\ I_M{}^A\ E_A{}^M - \int d^8z\ J_M{}^A{}_B\ \phi^M{}_A{}^B - i\int d^8z\ \eta^L c_L - i\int d^8z\ \rho^A b_A$$

If all equations Eq. (6) can be inverted,

$$\Omega = \Omega[\varphi, K_M{}^A, K''_M{}^A{}_B, L_M],$$
$$\Omega \equiv \Big(I_M{}^A, J_M{}^A{}_B, \eta_L, \rho_A\Big), \quad \varphi \equiv \Big(E_A{}^M, \phi^M{}_A{}^B, c^L, b^A\Big).$$

the effective action can be defined in terms of new variables, $\Gamma = \Gamma[\varphi, K_M{}^A, K''_M{}^A{}_B, L_M]$. Hence the following equalities hold:

$$\frac{\delta\Gamma}{\delta E_A{}^M} = -I_M{}^A, \quad \frac{\delta\Gamma}{\delta\phi^M{}_A{}^B} = -J_M{}^A{}_B, \quad \frac{\delta\Gamma}{\delta K_M{}^A} = -\frac{\delta W}{\delta K_M{}^A}, \tag{7}$$
$$\frac{\delta\Gamma}{\delta K''_M{}^A{}_B} = -\frac{\delta W}{\delta K''_M{}^A{}_B}, \quad \frac{\delta\Gamma}{\delta c^L} = i\eta_L, \quad \frac{\delta\Gamma}{\delta b^A} = i\rho_A, \quad \frac{\delta\Gamma}{\delta L_M} = -\frac{\delta W}{\delta L_M}.$$

If the transformation Eq. (5) is made in the path integral Eq. (4) one obtains (as the result of the invariance of the integral Eq. (4) under a change of variables) the Slavnov–Taylor (ST) identity (up to dependent on β terms):

$$\Bigg[\int d^8z\ I_M{}^A \frac{\delta}{\delta K_M{}^A} + \int d^8z\ J_M{}^A{}_B \frac{\delta}{\delta K''_M{}^A{}_B} - \int d^8z\ i\eta_M\left(\frac{\delta}{\delta L_M}\right)$$
$$+ \int d^8z\ i\rho^A\left(\frac{1}{\alpha}\partial_M \frac{\delta}{\delta I_M{}^A}\right)\Bigg] W = 0,$$

or, taking into account the relations Eq. (7), we have

$$\int d^8z\ \frac{\delta\Gamma}{\delta E_A{}^M}\frac{\delta\Gamma}{\delta K_M{}^A} + \int d^8z\ \frac{\delta\Gamma}{\delta\phi^M{}_A{}^B}\frac{\delta\Gamma}{\delta K''_M{}^A{}_B} + \int d^8z\ \frac{\delta\Gamma}{\delta c^M}\frac{\delta\Gamma}{\delta L_M}$$
$$-\int d^8z\ \frac{\delta\Gamma}{\delta b_A}\left(\frac{1}{\alpha}\partial_M E_A{}^M\right) = 0. \tag{8}$$

In addition to ST identity also there is the ghost equation that can be derived by shifting the antighost field b by an arbitrary field $\varepsilon(z)$ in the path integral. The consequence of invariance of the path integral with respect to such a change of variable is (in terms of the variables (6)) [2]

$$\frac{\delta\Gamma}{\delta b^A(z)} + \partial_M\ \frac{\delta\Gamma}{\delta K_M{}^A(z)} = 0. \tag{9}$$

The ghost equation (9) restricts the dependence of Γ on the antighost field b and on the external source K_M to an arbitrary dependence on their combination

$$\partial_M \; b^A(z) + K_M{}^A(z). \tag{10}$$

Starting with this point we can use method proposed in Refs. [4, 5, 6] for searching solution to Slavnov–Taylor identity in theories with local gauge symmetries. The main idea of Refs. [4, 5, 6] is to take a solution in which 1PI Lcc correlator of the theory is invariant itself with respect to Slavnov–Taylor identity. The $Lcc\phi_M$ and $LccE_M$ correlators have more weak behaviour in space of momentums and their contribution to the ST identity will be more weak in comparison with Lcc contribution. This results in invariance of the effective action which is local construction written in terms of dressed effective fields with respect to the BRST symmetry of diffeomorphisms (5) written also in terms of dressed effective fields.

At the same time, there is another symmetry for the structure group. It has been analyzed in Refs. [4, 5, 6] for the case of Yang–Mills theory and can be repeated here without modifications since all the constructions for the spin connection just repeat the analogous construction for the Yang–Mills connection. The field of vielbein participates in that symmetry as well as another matter field. According to the lines of that approach, the effective action in supergravity theory that is in consideration is the following:

$$\begin{aligned}\Gamma[\pi^\omega, E_M{}^A, \phi_{MA}{}^B, b, c, b', c'] &= -\frac{6}{\kappa^2}\int \; d^4y \; d^2\Theta \; \mathcal{E}(\tilde{E}_M{}^A, \tilde{\phi}_{MA}{}^B) R(\tilde{E}_M{}^A, \tilde{\phi}_{MA}{}^B) + \ldots \\ -\int \; d^8z \; \frac{1}{2\alpha}\left(\partial_M E_A{}^M\right)^2 &- \int \; d^8z \; \mathrm{Tr}\left(\frac{1}{\beta}\left[\partial_M \phi^M(z)\right]^2\right) - \int \; d^8z \; i \; \tilde{b}^A \; \partial_M \; \mathcal{L}_{\tilde{c}}\tilde{E}_A{}^M \\ &- \int \; d^8z \; 2 \; \mathrm{Tr} \; \left(\; i \; \tilde{b}'(z)\partial_M \; \tilde{\mathcal{D}}^M \; \tilde{c}'(z)\right) + \int \; d^8z \; \tilde{E} \; \pi^\omega \tilde{\mathcal{C}}^\omega,\end{aligned} \tag{11}$$

where all auxiliary fields K and L are set equal to zero. As one can see, the physical part of the effective action is local BRST invariant with respect to both local symmetries (Lorentz and diffeomorphism). Physical part starts with the first term of (11) and contains all other possible invariants in terms of chiral density, chiral superfield R, vielbein and spin connection. It is unclear at present how to derive exact form of the physical part. Dressed fields in the effective action are the effective fields convoluted with unspecified dressing functions,

$$\begin{aligned}\tilde{E}_M{}^A(z) &= \int \; d \; z' \; G_E^{-1}(z-z') \; E_M{}^A(z'),\\ \tilde{\phi}_{MA}{}^B(z) &= \int \; d \; z' \; G_\phi^{-1}(z-z') \; \phi_{MA}{}^B(z'),\\ \tilde{c}(z) &= \int \; d \; z' \; G_c^{-1}(z-z') \; c(z'),\\ \tilde{b}(z) &= \int \; d \; z' \; G_E(z-z') \; b(z').\end{aligned}$$

In the effective action (11) we have not done the integration yet over the Lagrange multiplier π. In 1PI diagrams this factor can be considered as background superfield. By requiring correspondence to the classical action we obtain that the constraint (2) has been modified to

the following form:

$$\pi^{\omega}\left(T-\tilde{\mathcal{D}}\tilde{E}\right)^{\omega}.$$

Integration over π in the path integral means that we have to resolve this constraint in the effective action. In comparison with the classical action we can derive the analogous solution with the only difference that instead of classical fields of the vielbein and spin connection we have to resolve it for the dressed vielbein and the dressed spin connection.

We comment here that this action should be considered only as one of the models for quantum supergravity. We do not pretend in this note for strict argument in favor of this action. However, this idea to write the effective action in terms of *dressed* (convoluted with some unspecified dressing functions) effective fields seems plausible physically and probably has natural physical interpretation.

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